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Outperformance Options

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The Outperformance Option’s Value in B-Shares

Suppose you live in B-Land where the currency is the B-share. In this country, the value of one share of stock A in B-shares is \( A_B(t) \equiv A_S(t)/B_S(t) \). The value of one B-share in dollars at \( t=0 \) was $1. The value of one B-share in B-Land is \( B_B(t) \) and is always equal to 1. The riskless interest rate at which you can earn interest on your B-shares is B’s dividend rate \( d_B \). The outperformance call in Equation 2 has a terminal payoff in B-shares given by dividing Equation 2 by \( B_S(T) \):

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Table 1 displays the A-hedge plus the B-hedge that together remove all exposure of the outperformance option to movements in A and B. Notice that the net position value of the totally hedged portfolio is zero.

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As long as the scaling property of the previous section holds, this general decomposition is still valid, except that the \( BS() \) function must be replaced by the American option value \( AM() \):
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\textsuperscript{1} The Value of An Option to Exchange One Asset for Another, William Margrabe, Journal of Finance, Vol XXXIII no. 1, 177-186, March 1978.
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